GRAVITY-MATTER ENTANGLEMENT IN REGGE QUANTUM GRAVITY

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Abstract

We argue that Hartle-Hawking states in the Regge quantum gravity model generically contain non-trivial entanglement between gravity and matter fields. Generic impossibility to talk about "matter in a point of space" is in line with the idea of an emergent spacetime, and as such could be taken as a possible candidate for a criterion for a plausible theory of quantum gravity. Finally, this new entanglement could be seen as an additional "effective interaction", which could possibly bring corrections to the weak equivalence principle.

The Model

Regge quantum gravity with scalar matter field:

$$\prod_{\epsilon=1}^{E}\int_{\mathbb{R}_0^+} dL_\epsilon\, \mu(L_\epsilon) \prod_{\sigma=1}^{V}\int_{\mathbb{R}} d\Phi_\sigma$$

$$\sum_{\Delta=1}^{F} A_\Delta(L)\Theta_\Delta(L)$$

$$\frac{1}{2}\sum_{\sigma=1}^{V} V_\sigma(L)\Big[g^{\mu\nu}(L)\Phi_{,\mu}\Phi_{,\nu} + U(\Phi)\Big]$$

$$Z_T = \int \mathcal{D}L \int \mathcal{D}\Phi \;\; \exp\Big[iS_{\text{Regge}}(L) + iS_{\text{matter}}(L,\Phi)\Big]\,.$$

The state sum is defined over a piecewise-linear spacetime manifold $\mathcal{M}_4$ corresponding to some particular triangulation T. The triangulation consists of vertices v, edges ϵ, triangles Δ, tetrahedra τ and 4-simplices σ.

The fundamental variables in the theory are the length $L_\epsilon \in \mathbb{R}_0^+$ of each edge, and the magnitude of the scalar field $\Phi_\sigma \in \mathbb{R}$ inside each 4-simplex.

For a review of the Loop Quantum Gravity framework see [1], and for the path integral approach (spinfoam models) see [2]. The spincube model is a categorical generalization of spinfoam models [3], and in one particular implementation it is equivalent to the Regge quantum gravity model [4].

The Wavefunction

A generic state:

Element of $\mathcal{H}_{\text{phys}}$

Choose coefficients such that $|\Psi\rangle \in \mathcal{H}_{\text{phys}} \subset \mathcal{H}_G \otimes \mathcal{H}_M$

Element of $\mathcal{H}_G \otimes \mathcal{H}_M$

$$|\Psi\rangle = \int \mathcal{D}l \int \mathcal{D}\phi \;\; \Psi(l,\phi) \;\; |l\rangle \otimes |\phi\rangle\,.$$

Every physical state must be invariant under the gauge symmetry $Diff(\mathcal{M}_4)$, which selects a subset $\mathcal{H}_{\text{phys}}$ out of the big Hilbert space $\mathcal{H}_G \otimes \mathcal{H}_M$ of all kinematically possible states. Among the gauge invariance conditions, the most complicated one is the scalar constraint $\hat{H}|\Psi\rangle = 0$, sometimes called Wheeler-DeWitt equation. Its solutions are very hard to find, but one well-known solution is called the Hartle-Hawking state [5].

The Hartle-Hawking state:

$$\Psi(l,\phi) = \Psi_{\text{HH}}(l,\phi) \equiv \int \mathcal{D}L \int \mathcal{D}\Phi \;\; \exp\Big[iS_{\text{Regge}}(L,l) + iS_{\text{matter}}(L,\Phi,l,\phi)\Big]\,.$$

Hartle-Hawking state is equal to a state sum $Z_T(l,\phi)$ over a manifold with a boundary. It is normalized as $\langle\Psi|\Psi\rangle = Z_{T\cup\bar{T}} = 1$.

Fields in the bulk

Fields on the boundary

Gravity-Matter Entanglement

Reduced density matrix:

$$\int \mathcal{D}l \;\; \langle l|\Psi\rangle \otimes \langle\Psi|l\rangle$$

$$Z_{T\cup\bar{T}}(\phi,\phi')$$

$$\hat{\rho}_M = \text{Tr}_G\,|\Psi\rangle \otimes \langle\Psi| = \int \mathcal{D}\phi \int \mathcal{D}\phi' \;\; \Big[\int \mathcal{D}l \;\; \Psi_{\text{HH}}(l,\phi)\Psi^*_{\text{HH}}(l,\phi')\Big] \;\; |\phi\rangle \otimes \langle\phi'|\,,$$

Entanglement:

$$\text{Tr}_M\,\hat{\rho}_M^2 = \int \mathcal{D}\phi \int \mathcal{D}\phi' \;\; \big|Z_{T\cup\bar{T}}(\phi,\phi')\big|^2 \neq 1\,.$$

RESULT !!!

$$\int \mathcal{D}\varphi \;\; \langle\varphi|\hat{\rho}_M^2|\varphi\rangle$$

By definition, a pure bipartite state is entangled if and only if it cannot be written as a product of pure states of two subsystems. To show that the state is generically entangled, we use a simple criterion for the purity of a general mixed state: a state $\hat{\rho}$ is pure if and only if $\text{Tr}\,\hat{\rho}^2 = 1$ [6].

Consequences

- In [7] Penrose argues that gravity-matter entanglement is at odds with (classical) spacetime, seen as a (four-dimensional) differentiable manifold. In light of this, our result could be seen as a quantitative indicator that in quantum gravity one cannot talk of "matter in a point of space", i.e., this result could be seen as a confirmation of a "spacetime as an emergent phenomenon".
- Thus, generic gravity-matter entanglement could be seen as a possible candidate for a criterion for a plausible theory of quantum gravity.
- Entanglement is in standard quantum mechanics a generic consequence of the interaction. This new entanglement can be regarded as a consequence of an effective interaction (such as the "exchange interactions", which are a consequence of quantum statistics). This additional "effective interaction" can potentially lead to corrections to the weak equivalence principle.

References

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