COSINE PROBLEM AND ANTIGRAVITY IN EPRL/FK SPINFOAM MODEL

Marko Vojinović
Group of Mathematical Physics, University of Lisbon

EPRL/FK MODEL

Short description of EPRL/FK model of quantum gravity:

- Covariant approach to the formulation of quantum gravity.
- The path integral is defined as

$$Z_\sigma = \sum_c \prod_{f\in\sigma} A_f(c) \prod_{e\in\sigma} A_e(c) \prod_{v\in\sigma} A_v(c).$$

- σ denotes a 2-complex dual to a spacetime triangulation, f, e, v count the faces, edges and vertices of the 2-complex,
- colors c are $SU(2)$ spins on each face, $j_f \in \mathbb{N}_0/2$, and $3D$ unit vectors $\vec{n}_{ef} \in S^2$ for every pair ef,
- amplitudes are

$$A_f = 2j_f + 1, \qquad A_e = 1, \qquad A_v = W_v^{\mathrm{EPRL/FK}}.$$

EPRL/FK MODEL

Main properties of the model:

- Spectrum of the area operator,

$$A_f = 8\pi\gamma l_p^2 \sqrt{j_f(j_f+1)},$$

where A_f is the area, l_p is the Planck length, γ is the Barbero-Immirzi parameter.

- The classical limit,

$$\frac{1}{8\pi\gamma}\frac{A}{l_p^2} = \sqrt{j(j+1)} \approx j \gg 1.$$

- Asymptotics of the vertex amplitude in the limit $j \to \infty$,

$$W_v(j,\vec{n}) \approx N_+(j)e^{i\gamma S_v(j)} + N_-(j)e^{-i\gamma S_v(j)}, \qquad \text{//looks like } \cos(S_v)\text{//}$$

where $S_v(j)$ is the area-Regge action for one 4-simplex dual to the vertex v.

EPRL/FK MODEL

Coupling of matter fields:

- We redefine the vertex amplitude,

$$A_v(j_f, \vec{n}_{ef}, \phi_r) = W_v(j_f, \vec{n}_{ef}) e^{iS_v^{\text{matter}}(j_f, \vec{n}_{ef}, \phi_r)},$$

  where ϕ_r are matter fields, r counts all degrees of freedom for all matter fields, S_v^{matter} is the matter action for one 4-simplex dual to the vertex v.

- The path intgeral,

$$Z_\sigma = \sum_j \int \prod_{ef} d\vec{n}_{ef} \int \prod_r d\phi_r \prod_f [2j_f + 1] \prod_v W_v(j, \vec{n}) e^{iS_v^{\text{matter}}(j, \vec{n}, \phi)}.$$

- If we "freeze out" gravitational degrees of freedom, we have

$$Z_\sigma \sim \mathcal{N} \int \prod_r d\phi_r \prod_v e^{iS_v^{\text{matter}}(j, \vec{n}, \phi)} \sim \mathcal{N} \int \mathcal{D}\phi \; e^{iS^{\text{matter}}[g, \phi]}.$$

EPRL/FK MODEL

The cosine problem:

- What we would like to have (in some suitable limit):

$$W_v \sim e^{iS_v},$$

so that

$$Z_\sigma \sim \sum_j \int d\vec{n} \int d\phi \prod_v e^{i(S_v + S_v^M)} \sim \int dj \int d\vec{n} \int d\phi \; e^{\sum_v S_v^{GM}} \sim \int \mathcal{D}g \int \mathcal{D}\phi \; e^{iS}.$$

- What we do have:

$$W_v \sim e^{iS_v} + e^{-iS_v} \sim \cos(S_v),$$

so that

$$Z_\sigma \sim \sum_j \int d\vec{n} \int d\phi \prod_v \cos(S_v) e^{iS_v^M} \sim \int \mathcal{D}g \int \mathcal{D}\phi \prod_{\mathcal{M}} \left[e^{i(S+S^M)} + e^{i(-S+S^M)} \right].$$

EFFECTIVE ACTION

How do we compute the effective action in QFT:

- Consider a typical QFT,

$$Z[J] = \int \mathcal{D}\phi \; e^{iS[\phi]+i\int J\phi}.$$

- Effective action is defined as a Legendre transform

$$\Gamma[\phi] = -i \log Z[J[\phi]] - \int \phi J[\phi],$$

- from where one can derive a functional integrodifferential equation:

$$e^{i\Gamma[\phi]} = \int \mathcal{D}\tilde{\phi} \; e^{iS[\phi+\tilde{\phi}]-i\int \frac{\delta\Gamma}{\delta\phi}\tilde{\phi}}.$$

- The field ϕ is called the “background”.

EFFECTIVE ACTION

How do we compute the classical limit:

- We define the classical limit as

$$\phi \to \infty, \qquad S[\phi] \gg 1, \qquad\qquad //S[\phi] \gg \hbar//$$

- expand the effective action in an asymptotic series for this limit,

$$\varGamma = \varGamma_0 + \varGamma_1 + \varGamma_2 + \ldots, \qquad \varGamma_{n+1} = o(\varGamma_n),$$

- substitute all this into the functional integrodifferential equation

$$e^{i\varGamma[\phi]} = \int \mathcal{D}\tilde{\phi}\; e^{iS[\phi+\tilde{\phi}]-i\int \frac{\delta\varGamma}{\delta\phi}\tilde{\phi}},$$

- and we solve it perturbatively:

$$\varGamma = S + \frac{i}{2}\,\mathrm{tr}\log S'' + o(\log S).$$

EFFECTIVE ACTION

Multiple solutions for a classical limit:

- Type I — the limit $\phi \to \infty$ can be taken in multiple ways,

$$\begin{aligned} \phi = \phi_1 \to \infty \qquad &\Rightarrow \qquad \Gamma[\phi_1] = S_1[\phi_1] + \ldots, \\ \phi = \phi_2 \to \infty \qquad &\Rightarrow \qquad \Gamma[\phi_2] = S_2[\phi_2] + \ldots, \end{aligned}$$

  where ϕ_1, ϕ_2 are different "configurations" of the fields and S_1, S_2 different classical "regimes".

- Type II — the initial action can be of the form

$$S[\phi] = -i \log \left[e^{iS_1[\phi]} + e^{iS_2[\phi]} \right]$$

  so that the effective action equation has multiple solutions for one and the same field configuration,

$$\Gamma[\phi] = S_1[\phi], \qquad \Gamma[\phi] = S_2[\phi].$$

- If actions S_1 and S_2 give equivalent equations of motion, then the limit is the same. Otherwise, the classical limit does not exist. $// \lim_{x\to\infty} \sin(x) = ??//$

CLASSICAL LIMIT OF THE EPRL/FK MODEL

Effective action equation:

$$
\begin{aligned}
e^{i\Gamma(j,\vec{n},\phi)} \;=\; & \sum_{j'} \int \prod_{ef} d\vec{n}'_{ef} \int \prod_{r} d\phi'_r \; e^{-i\left(\sum_f \frac{\partial \Gamma}{\partial j_f} j'_f + \sum_{ef} \frac{\partial \Gamma}{\partial \vec{n}_{ef}} \vec{n}'_{ef} + \sum_r \frac{\partial \Gamma}{\partial \phi_r} \phi'_r\right)} \\
& \prod_f \left[2\left(j_f + j'_f\right) + 1\right] \prod_v W_v(j + j', \frac{\vec{n} + \vec{n}'}{\|\vec{n} + \vec{n}'\|}) e^{iS_v^{\mathrm{matter}}(j+j', \frac{\vec{n}+\vec{n}'}{\|\vec{n}+\vec{n}'\|}, \phi+\phi')}.
\end{aligned}
$$

Solutions in the classical limit $j = j(L), \vec{n} = \vec{n}(L)$ where $L, \phi \to \infty$:

$$
\begin{aligned}
\Gamma_+(L,\phi) &= \frac{1}{8\pi l_p^2} S^{\mathrm{Regge}}(L) + S^{\mathrm{matter}}(L,\phi), \\
\Gamma_-(L,\phi) &= -\frac{1}{8\pi l_p^2} S^{\mathrm{Regge}}(L) + S^{\mathrm{matter}}(L,\phi), \\
\Gamma_\epsilon(L,\phi) &= \frac{1}{8\pi l_p^2} \sum_v \epsilon_v S_v^{\mathrm{Regge}}(L) + S^{\mathrm{matter}}(L,\phi), \qquad \epsilon_v = \pm 1.
\end{aligned}
$$

CLASSICAL LIMIT OF THE EPRL/FK MODEL

The continuum limit:

$$S^{\mathrm{Regge}}(L) \to \frac{1}{2} S_{\mathrm{AH}}[e], \qquad S^{\mathrm{matter}}(L,\phi) \to S_{\mathrm{M}}[e,\phi].$$

Gravity solution:

$$\Gamma_{+}[e,\phi] = \frac{1}{16\pi l_p^2} S_{\mathrm{AH}}[e] + S_{\mathrm{M}}[e,\phi],$$

Antigravity solution:

$$\Gamma_{-}[e,\phi] = -\frac{1}{16\pi l_p^2} S_{\mathrm{AH}}[e] + S_{\mathrm{M}}[e,\phi],$$

"Intermediate" solutions:

$$\Gamma_{\epsilon}[e,\phi] = \frac{1}{16\pi l_p^2(x)} S_{\mathrm{AH}}[e] + S_{\mathrm{M}}[e,\phi], \qquad l_p^2(x) = \pm l_p^2.$$

CLASSICAL LIMIT OF THE EPRL/FK MODEL

How to fix these results?

- Redefine the gravitational sector so that

$$A_v \sim e^{iS_v^{\text{Regge}}}.$$

$\Rightarrow$ This is not the EPRL/FK model anymore!

- Redefine the matter coupling so that

$$A_v \sim \cos(S_v^{\text{Regge}} + S_v^{\text{materije}}).$$

$\Rightarrow$ Violates the equivalence principle!

$\Rightarrow$ Does not remove the "intermediate" limits!

- Postulate that the effective action is not a well defined notion in QG (!!!)

$\Rightarrow$ Why does it work in QFT so well?!

THANK YOU!